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Comment on Taylor's paper. by Max Shiffman July 1943

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During the contracting stage when the bubble is near its minimum size, the pressure at a specified point in the water is a maximum at or very near to the time when the bubble has its minimum size. To estimate the terms in Taylor's formula [29] for the pressure, write the equations [1], [2] at the instant when $\dot{a} = 0$. They are, ignoring all hydrostatic terms as negligible,

(i) $\dot{U} = 2g$

(ii) $\frac{1}{3}\pi\rho a^3 U^2 = W - G(a)$.

It is also necessary to have an expression for the term $(a^2 \dot{a})^{\cdot}$ occuring in [29] . It is obtained from the ~~equation~~ following equation, which can be derived from the Lagrangian equations of motion or from Taylor's equations [1] and [2] ,

(iii) $\frac{p(a)}{\rho} - gz + \frac{1}{2}\dot{a}^2 + \frac{1}{4}U^2 - \frac{(a^2\dot{a})^{\cdot}}{a} = 0$

where $p(a)$ is the pressure of the gas. Thus,

(iv) $(a^2 \dot{a})^{\cdot} = a\left(\frac{p(a)}{\rho} + \frac{1}{4}U^2\right)$

at the instant when $\dot{a} = 0$.

By substituting these results in Taylor's [29] , one sees that the dominant term is the first , and if a/r is about 1/4 or less (which is the case in Taylor's ~~xxxxxx~~ example) all the other terms are negligible . We may therefore set, with little error,

(v) $p = \frac{\rho(a^2 \dot{a})^{\cdot}}{r} = \frac{a}{r}\left(p(a) + \frac{1}{4}\frac{W-G(a)}{\frac{1}{3}\pi a^3}\right) = \frac{1}{r}\left(\frac{W}{\frac{4}{3}\pi a^2} - \frac{2-\gamma}{\gamma-1}a\,p(a)\right)$

where $p(a) = \text{constant}/a^{3\gamma}$.

One can easily determine the value of a which will maximize

the coefficient of $1/r$ in (v) . Using Taylor's equations [10], [12] for TNT , this value of a turns out to be

$$\text{(vi)} \qquad a = 1.30\, M^{1/3} \qquad \text{(for maximum possible } p \text{)}$$

where M is the mass of the explosive.

This value is to be compared with the least possible value of the minimum radius a , which would occur if there were no vertical motion of the bubble and is determined by $G(a) = W$. This value is

$$\text{(vii)} \qquad a = 1.25\, M^{1/3} \qquad (\text{ when } G(a) = W\text{ }) .$$

On the other hand, the minimum radius obtained by Taylor is considerably larger than this because of the vertical motion of the bubble. It is 3.3 times (vii) , or

$$\text{(viii)} \qquad a = 4.13\, M^{1/3} \qquad . \text{ (}$$

Of course all these results must be modified because the assumption of incompressibility is inadmissible near the minimum size of the bubble. The actual minimum radii are larger than those given by (vi), (vii), (viii) because of the acoustic radiation of energy. But (vi) - (viii) at least give the correct order of magnitude.

A comparison of (vi) with (vii) and (viii) shows that the upward motion of the bubble, although causing the contraction to take place nearer the target, prevents the bubble from attaining a much smaller radius and so produces a correspondingly weaker pressure pulse. We see that the most important factor in obtaining a high pressure pulse is to reach as small a radius as possible.

If one includes deviations from spherical symmetry in the course of the motion of the bubble, it is possible to show that the upward velocity of the bubble will be decreased. Also, equation (iii) will continue to hold (at least to a first order) so that a decrease in U will ~~xxxx xlx~~ yield a smaller minimum radius a.

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